

Advance in dynamical spontaneous symmetry breaking

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Abstract

Recently, a condition is derived for a nontrivial solution of the Schwinger-Dyson equation to be accompanied by a Goldstone bound state in a special quantum electrodynamics model. This result is extended and a new form of the Goldstone theorem is obtained in a general quantum field theory framework.

I. Introduction

Dynamical spontaneous symmetry breaking (DSSB) has been extremely useful in a wide variety of topics in particle physics, superconductivity theory (SCT), as well as condensed matter physics.¹⁻³

There are two aspects in the mechanism of DSSB, the mass generation of fermions and the existence of the Goldstone boson. More concretely, in particle physics, chiral symmetry breaking occurs when the self-consistent Schwinger-Dyson (SD) equation for the fermion mass develops a non-trivial solution as the coupling strength reaches a critical value. In addition, for the same coupling strength, there must also exist a bound state solution of the Bethe-Salpeter (BS) equation, with vanishing 4-momenta, corresponding to the massless pseudoscalar Goldstone boson. When both of these conditions are met, the self-energy solution shall be called a symmetry-breaking solution.

Recently, a criterion for a solution of the SD equation to signal a DSSB is deduced in a special quantum electrodynamics (QED) model.^{1,2} In this paper we extend this result to quite a general case. At the same time a new form of the Goldstone theorem is obtained in a general quantum field theory framework.

II. Goldstone theorem.

The Goldstone theorem has been established well. We know from it that a massless boson appears whenever a generator of a global continuous symmetry group is broken spontaneously. The quantum number of this particle corresponds to that of the operator of the current which is broken. For massless quantum field

theory, when the mass is generated due to the self-energy, the chiral symmetry is broken so that the Goldstone particle is a pseudoscalar boson. We associate this to a bound state solution of the BS equation, in which the fermion mass will be identified with the dynamically generated mass from the SD equation. In the following our discussions will be in this framework.

The renormalized fermion propagator will be written as $S_f^{-1}(p) = \gamma \cdot p \alpha(p^2) - \beta(p^2)$. The equation satisfied by the renormalization vertex function is⁴

$$\begin{aligned} \Gamma_\mu(p', p)_{\gamma\delta} &= Z_1(\gamma_\mu)_{\gamma\delta} - \int \frac{d^4 q}{(2\pi)^4} [S_f(p' + q)\Gamma_\mu(p' + q, p + q)S_f(p + q)]_{\beta\alpha} \times \\ &\quad \times K_{\alpha\beta, \delta\gamma}(p' + q, p + q, q) \end{aligned} \quad (1)$$

Where the $K_{\alpha\beta, \delta\gamma}(p' + q, p + q, q)$ is the fermion-antifermion two body irreducible function. Substitute it in the renormalized Ward's identity, $(p' - p)_\mu \Gamma_\mu(p', p) = S_f^{-1}(p') - S_f^{-1}(p)$ and suppose that the $K_{\alpha\beta, \delta\gamma}(p' + q, p + q, q)$ dependent on its argument only through the difference $q = p' - p$, we obtain:

$$S_f^{-1}(p)_{\gamma\delta} = Z_1(\gamma_\mu)_{\gamma\delta} + \int \frac{d^4 p'}{(2\pi)^4} S_f(p')_{\beta\alpha} \times K_{\alpha\beta, \delta\gamma}(p' - p) \quad (2)$$

Taking trace on both sides and similarly with the multiplication of $\gamma \cdot p$ before trace, the SD equation for the massless fermion self-energy is simplified into the following set of equations:

$$\beta(p^2) = \int \frac{dp'^4}{(2\pi)^4} \frac{\beta(p'^2)}{p'^2 \alpha^2(p'^2) + \beta^2(p'^2)} \times \text{Tr} [K(p' - p)] \quad (3)$$

$$\alpha(p^2) = 1 + \frac{1}{p^2} \int \frac{dp'^4}{(2\pi)^4} \frac{\alpha(p'^2)}{p'^2 \alpha^2(p'^2) + \beta^2(p'^2)} [(\gamma \cdot p)_{\gamma\delta} (\gamma \cdot p')_{\beta\alpha} K(p' - p)_{\alpha\beta, \delta\gamma}] \quad (4)$$

Here we have used a topological characteristic which leads to a conclusion that in the $K(p' - p)$ the γ_μ appears and only appears in pair. The trivial solution, $\beta(p^2) = 0$ and $\alpha(p^2) = \text{finite}$, corresponds to that of the symmetric vacuum. But it is certainly true that not all of the nontrivial solutions signal spontaneous symmetry breaking. The criterion that a nontrivial $\beta(p^2)$ does trigger DCSB is that there must be an accompanying Goldstone boson. That is to say, there must be a bound state solution of the BS equation, with the appropriate quantum numbers. We express the wave function for a bound state composed of the fermion-antifermion pair of the same kind as $\chi_k(p^2)$. The BS equation is⁵

$$\left[S_f^{-1}\left(\frac{k}{2} + p\right) \chi_k(p^2) S_f^{-1}\left(\frac{k}{2} - p\right) \right]_{\gamma\delta} = \int \frac{d^4 p'}{(2\pi)^4} \chi_k(p^2)_{\beta\alpha} \times K_{\alpha\beta, \delta\gamma}(p' - p) \quad (5)$$

Since the Goldstone state has $\ell^P = 0^-$ and vanishing mass, we take $k = 0$ and write $\chi_k(p) = \chi_0^P(p^2) \times \gamma_5$. Now we put forward an crucial step by requiring the fermion-antifermion two body irreducible function, $K(p' - p)$, to satisfy the

relation, $\gamma_{5\beta\alpha}K(p' - p)_{\alpha\beta,\delta\gamma} = -[K(p' - p)\gamma_5]_{\gamma\delta}$. In the same steps as we do to the SD equation, it leads to the important result:

$$[p^2\alpha^2(p^2) + \beta^2(p^2)]\chi_0^P(p^2) = \int \frac{dp'^4}{(2\pi)^4} \chi_0^P(p'^2) \times T_r [K(p' - p)] \quad (6)$$

Introduce the function $\psi(p^2) = [p^2\alpha(p^2) + \beta^2(p^2)]\chi_0^P(p^2)$, then eq.(6) becomes:

$$\psi(p^2) = \int \frac{dp'^4}{(2\pi)^4} \frac{\psi(p'^2)}{p'^2\alpha^2(p'^2) + \beta^2(p'^2)} \times T_r [K(p' - p)] \quad (7)$$

If there is a solution of eq.(3), then we have a solution of eq.(7) as $\psi(p^2) = c\beta^2(p^2)$. c is a arbitrary constant. It give a solution of the wave function of the Goldstone boson. Thus we obtained a new form of the Goldstone theorem here.

III. Condition on the symmetry breaking solution.

For χ_0^P to be a wave function of a Goldstone boson which accompanies with the non-trivial solution of the SD equation, it must satisfy a normal condition:⁵

$$\int dq^4 [\alpha^2(q^2)q^2 + \beta^2(q^2)] |\chi_0^P(q^2)|^2 = \text{finite and nonzero} \quad (8)$$

In order for DSSB to occur, eq.(3) must have a non-trivial solution $\beta(p^2) \neq 0$. However, this is a necessary, but not a sufficient condition. For $\beta(p^2)$ to be symmetry-breaking solution, it must satisfy also an additional normalization condition which is obtained from the normal condition of BS equation. It can be proved that the necessary and sufficient condition for a non-vanishing solution $\beta(p^2)$ of eq.(3) to be symmetry-breaking is that it must satisfy, together with the finite solution $\alpha(p^2)$ of eq.(4), the condition:

$$\int dq^4 \frac{1}{q^2\alpha^2(q^2) + \beta^2(q^2)} |\beta(q^2)|^2 = \text{finite and nonzero}. \quad (9)$$

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